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A Schema for Transitioning from Rectilinear Coordinate Spaces to Pseudo-Hexagonal Spaces: Help Me Rhombus, Help, Help Me Rhombus

There are two main approaches to managing wargame units interactions with their environments (movement, line of sight, area of effect, etc.): freespace and gridded. In the freespace approach, the units exist as entities in a continuous volume of (usually) Cartesian 3D space. They move in any direction (based on interaction with “terrain” that occupies the same space) and interact with each other based on references and displacements from their position in that space. In the gridded approach, space is broken up into (usually regular) shaped pieces. Units are considered to occupy the entire volume of one of these pieces; movement, line of sight, and other interactions are based on the relationships among the spaces rather than the absolute positions of the units themselves. Both approaches have advantages and drawbacks.

The freespace approach seems to be more “natural” or “realistic” from an intuitive sense. Units can occupy a number of positions and orientations that is only limited by the degree of effort you are willing to put into measurement. It is easy to make analogies between the physical world and the wargame battlespace.

The challenges with freespace are mainly in complexity and level of effort in implementation. Calculating relationships among points and volumes in freespace can be difficult, especially when dealing with paths through that space that are non-linear (they “go around” things in the space). There are a lot of physical techniques that can alleviate these challenges, mostly due to the close correspondence to the physical world. These techniques, however, usually increase the level of effort challenges inherent in the freespace approach. The archetypical level of effort challenge puts the player in the position that since they have so many degrees of freedom in action, that to gain best advantage in those actions, they have to put large amounts of effort into examining each action in a high level of detail. For example, an individual combatant who is walking a “S” path in freespace can have a significantly different advantage of position by making minute changes to the curvature and inflection points in the path.

The gridded approach is generally seen to overcome the challenges of freespace. By dividing the space up into reasonably sized discrete *cells*, movement, line of fire, cover, and other interactions can be implemented with relatively simple systems based on whole number counting. Additionally, since there are no different positions within the cells, there are fewer degrees of freedom in actions, and thus the required or desired level of effort (“Wait a minute! If I moved my guy a half a millimeter to the right, would that let me ...”) in considering options is lessened. This decreased tactical focus frees up players’ cognitive resources for other, generally considered “deeper,” considerations such as “look-ahead,” strategy, and consideration of the meta-game.

These advantages do come at a cost. The reduction in degrees of freedom equates to a reduction in verisimilitude. For example, units can occupy a certain cell or another adjacent cell, but cannot occupy a space “in between” the cells that might logically

equate to a tactical advantage over either, or be the outcome of a more naturalistic decision process such as the desire to stop movement directly “behind” a pillar that acts as an obstruction to fire rather than “to the right” or “to the left” of it. It is frequently the case that in three dimensional gridded systems one of the dimensions (usually “altitude”) is treated differently than the other two (forming the “ground”). This usually introduces complexity (in the form of additional rules) and again detracts from the verisimilitude.

There are other, more basic problems with the “feel” of gridded systems from the constraints imposed by the cell structure. In square grids moving four units *north* and then three units *east* results in taking seven unit steps, but the measured distance between the centers of the starting and ending cells is only five units. Breaking this movement up into a sequence of smaller movements does not alleviate the problem.

Hexagonal grids can mitigate this problem with square grids, as the correspondence between the minimum number of steps between cells and the measured distance is much closer. Additionally, hexagonal grids provide six degrees of freedom in movement vice the four of square grids at no increase in rule complexity. Even though there are more options, they tend to seem less “natural,” possibly because people are more familiar with rectilinear grids and have more patterns of mind based on “forward, backward, right, left” as opposed to “every sixty degrees.” The biology of bilateral symmetry may even have an effect on this, which would be a difficult part of the mindset to overcome.

An additional issue with hexagonal grids is that they are not closed under composition, that is, you cannot take a regular hexagon and break it up into any whole number of smaller hexagons that completely cover the same shape. Square grids, however, are easily decomposed into smaller square grids, or aggregated into larger ones.

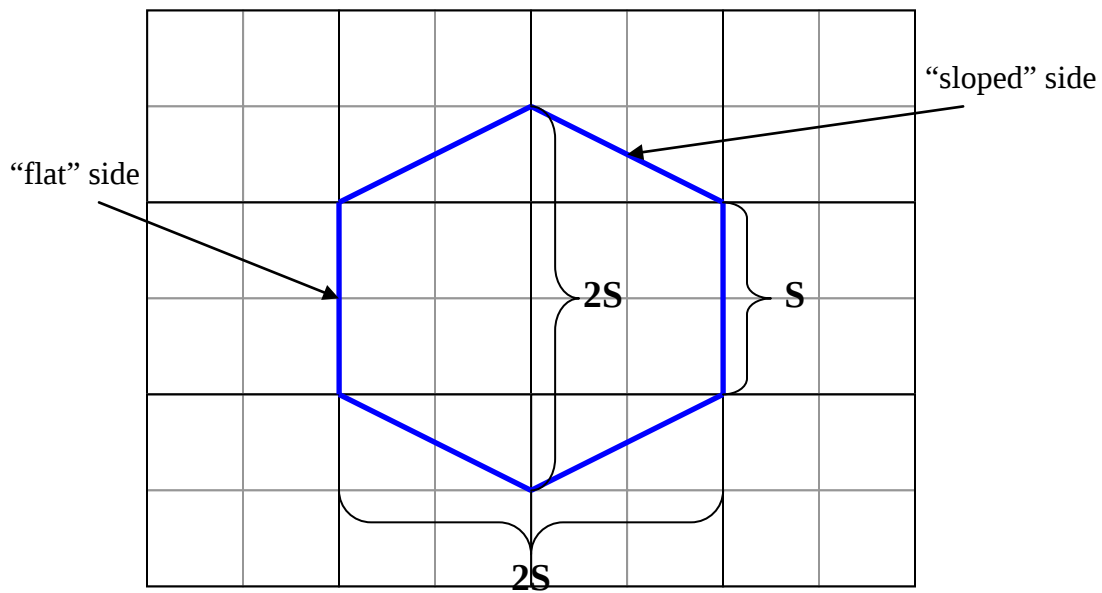
The general issue that this discussion has addressed is that there is no “perfect” approach to implementing a wargaming battlespace. Each of them (and this extends to others not discussed) has different sets of advantages and disadvantages. Nothing will change that basic nature of the various approaches, nor would it be desirable to do so. Along with the advantages, the challenges define the feel of the game and focus the thinking of the players on certain aspects and away from others.

Directing players’ thinking and behavioral patterns is a very important factor in the enjoyment, catharsis (immersion), and experiential learning value of wargames. While the approach to implementing the battlespace is not the only driver of the players’ experiences, it is a significant one since it affects most decisions and other thinking about the game. The goal of the proposed schema is to provide an approach to integrating square and hexagonal grids to make a hybrid battlespace that will allow the game to provide players access to the feel of either system, in portions where appropriate, with a minimum of difficulties in transition between the two.

So what are the general feels of the square and hexagonal systems and what is a reasonable use case where you would want both combined?

The square gridded environment provides a *best fit* for man made environments, especially the interiors of structures and built up urban areas. The rectilinear structure of streets and hallways can usually be lined up with the grid itself, creating a natural harmony between the referent and the representation. In general, the more densely built up a man made environment is, the less opportunity there is for “diagonal” movement, so the less important the challenges with moving large distances at angles not aligned with the grid are. This system still has significant challenges representing large open areas and structures with multiple grids, offset angularly from each other.

As a sort of compliment, the hexagonal grid is most popular with outdoor environments. It provides the additional degrees of freedom of motion expected in open areas and has much less separation between steps and measured distances over larger portions of the environment. Likewise, the hexagonal system has significant challenges representing rectilinear shapes. In general, they have to be constrained to unnatural, offset positions and the hexes that contain the edges of the rectilinear shape usually have some *loss* – large portions of area that are inaccessible to the units in the game, but not actually occupied by the represented terrain itself.



Our stretched hexagon

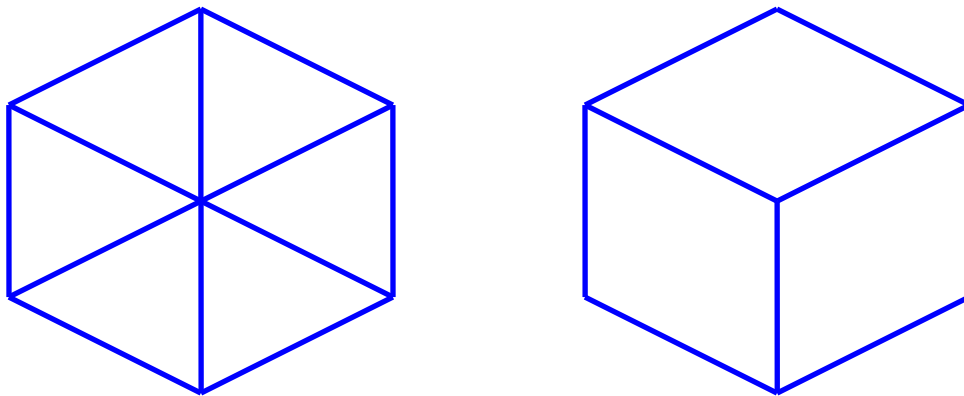
One of the most common methods to bring these two approaches together is to not bring them together. Or, more specifically, to conduct different parts of an overall scenario on different maps that have entirely one or the other type of grid, as appropriate. The second most popular method is to severely limit one or the other type of terrain on the map and allow it to be represented with the approach appropriate for the dominant part of the terrain. The entry point to the city might have a small strip of outdoor terrain at one edge of the map or the number houses and outbuildings in a rural area might be minimized. Both of these methods work well enough, but are certainly compromises to the idea of a hybrid square and hexagonal grid. The easy way, just mashing the two grids together and

accepting or artificially putting obstructions in the inconvenient places where the two are not compatible is also somewhat unsatisfying.

The proposed approach to combining square and hexagonal approaches, which we will call the rhombus interface, leverages rhombuses constructed from equilateral triangles into which the hexagon can be decomposed to bridge the gap between the approaches, maintain relative consistency between the two as much as possible, and provide most of the feel of the hexagonal approach.

The approach is based on a chosen square grid, with sides of the cells being length s . The hexagonal grid to be integrated will be made of slightly “stretched” hexagons (not quite regular hexagons), adjusted to mesh with a square grid of side $s/2$. One pair of opposite sides for these hexagons will be of length s and parallel to one pair of sides of the square grid and spaced $2s$ apart from each other, with end points aligned. We will call these the *flat* sides. The point-to-point diagonal of the hexagon between the flat sides will also be of length $2s$, and will be in the middle of the flat sides, centered on their length. The sides of the hexagon that connect the flat sides to this diagonal will be called the *sloped* sides.

Note that by arranging one of the flat sides with one side of the square of the base grid, the points of the point-to-point diagonal that defines the sloped sides align with the half grid. Also note that the area of this hexagon is exactly $3s^2$, or the area of three of the full squares of the base grid.

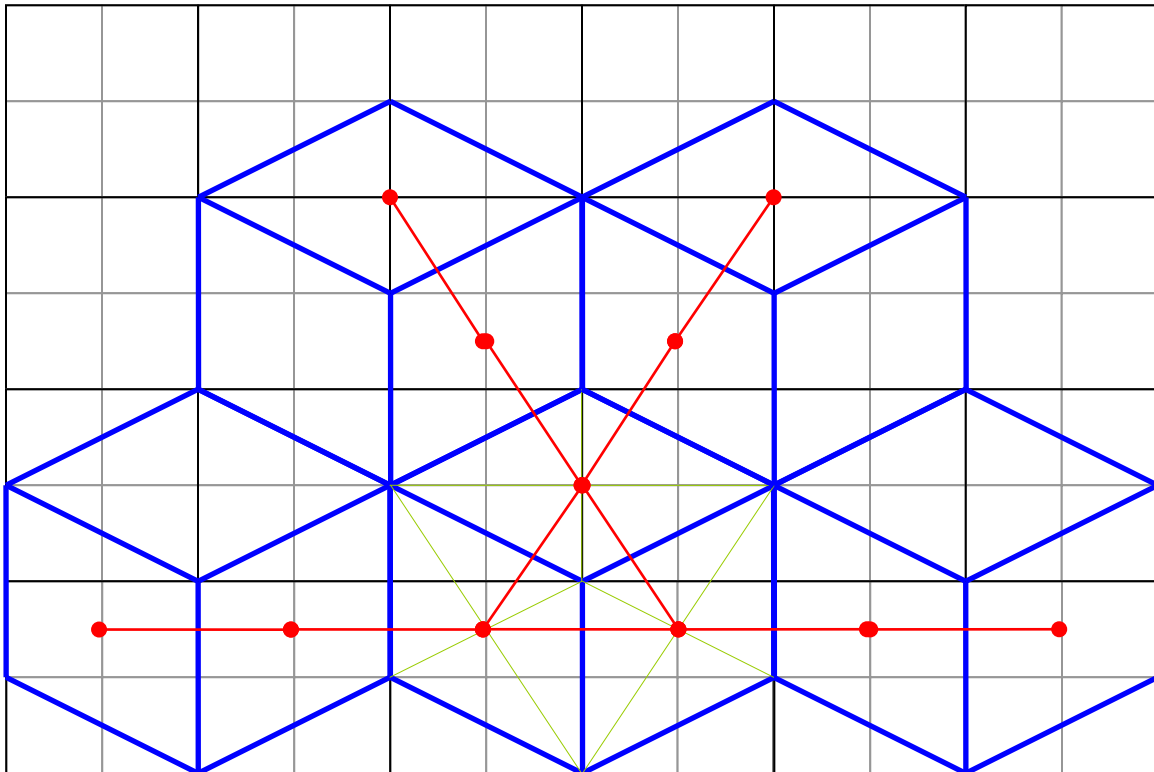


triangles and a top set of rhombi

The vertices of the hexagon align with a whole number ratio of the base grid, with no need to approximate irrationals. The half-unit is a standard subdivision of both English and Metric measures and fairly easy to physically approximate without measuring. This makes calculation, vector construction, graphical representation, hand drawing of the grid, and calculation of an individual hexagon from a reference point without calculation of all the intervening hexagons all relatively easy and computationally simple. These properties will propagate across the grid as the hexagons are repeated.

Next, we will break this hex down into six triangles, then aggregate them to make the rhombi that will be the base cell for the organic part of the grid. Dividing the hexagon is simply connecting the vertices with the center. Creating the rhombi is likewise a simple matter of choosing pairs of triangles that share a side and eliminating that side. The resulting four sides make a rhombus. There are two different ways you can do this for each hexagon. The one illustrated above will be called a *top* set, the opposite will be called a *bottom* set.

Since each of the three rhombi are congruent (each is composed from a pair of congruent triangles), it is easy to see that each has an area one-third of the original hexagon, or s^2 , the same area as the squares in the base grid. Also, the lengths of the pairs of sides are s for the flat sides (parallel to the flat hexagon sides) and a little less than $1.2s$ ($\sqrt{5}/2 s$, to be precise) for the sloped slides (again, parallel to the sloped sides of the hexagon).



cell to cell transitions

The rhomboid cell, then is decently close in basic size and shape parameters to the square grid cell and fairly simple to construct from that underlying grid. These are nice descriptive parameters, but the real key performance parameters are how the grid supports interactions among units in different cells.

Qualitatively, we can see that the rhomboid grid gives us similar degrees of freedom for cell to cell transition as the square grid – four choices by passing into an adjacent rhombus that shares a flat (in some cases) or a sloped (in all cases) side. In the aggregate,

however, the cell to cell transition is similar to the hexagonal system, giving us six “straight line” directions overall.

Quantitatively, the cell to cell distance transitioning through a flat side is exactly s . The cell to cell distance transitioning through a sloped side is $\sqrt{(13/16)}s$, or a little over $.9s$, for just less than a 10% loss for steps vice geometric distance. When you consider the effect on multiple transitions that change direction, the variances get larger, but also become dependent on path selection to get from one cell to another. A good metric to evaluate the overall distortion over larger distances is to calculate the relative area of a geometric circle of radius x and compare it to the sum of the areas of the unique cells that can be reached by x cell to cell transitions.

steps	circle area	square grid area		hexagonal grid area		rhomboid grid area	
1	3.14	5	159.15%	6.06	192.97%	5	159.15%
2	12.57	13	103.45%	16.45	130.94%	13	103.45%
3	28.27	25	88.42%	32.04	113.33%	27	95.49%
4	50.27	41	81.57%	52.83	105.10%	43	85.55%
5	78.54	61	77.67%	78.81	100.34%	65	82.76%
6	113.10	85	75.16%	109.99	97.25%	93	82.23%
7	153.94	113	73.41%	146.36	95.08%	123	79.90%
8	201.06	145	72.12%	187.93	93.47%	161	80.07%
9	254.47	181	71.13%	234.69	92.23%	199	78.20%
10	314.16	221	70.35%	286.65	91.24%	247	78.62%
11	380.13	265	69.71%	343.81	90.45%	293	77.08%
12	452.39	313	69.19%	406.17	89.78%	351	77.59%
13	530.93	365	68.75%	473.72	89.22%	405	76.28%
14	615.75	421	68.37%	546.46	88.75%	473	76.82%
15	706.86	481	68.05%	624.40	88.34%	535	75.69%
16	804.25	545	67.77%	707.54	87.98%	613	76.22%
17	907.92	613	67.52%	795.88	87.66%	683	75.23%
18	1017.88	685	67.30%	889.41	87.38%	771	75.75%
19	1134.11	761	67.10%	988.13	87.13%	849	74.86%
20	1256.64	841	66.92%	1092.06	86.90%	947	75.36%

Because all these approximations converge on their own “best fit” to a circle, you can make any of them come arbitrarily as close as you want by changing the area of the grid cell. However, the point of the comparison is to evaluate the grids that also have a good fit to the “one unit step” from center cell to center cell. The square and rhomboid used above are the ones discussed. The hexagonal grid used is the one with a flat to flat diameter of s , which is the one that gives a center cell to center cell transition of s . At long ranges, the rhomboid approach ends up converging on an approximation of a circle somewhat better than the square, but not quite as good as the hexagonal.

Based on the above analysis, the rhomboid grid is *almost* as good as the hexagonal in several ways, so why not just use the hexagonal grid? The answer comes in trying to provide a reasonable interface between the two grids. A regular hexagon will either have at least one side length or one of the diameters that is an irrational value, which means

only certain integer multiple numbers of rows and columns of hexagons will make a good fit with a square grid. Regardless of where those convergence points are for any two particular schema, there will still be significant loss where they are incompatible. Additionally, in the interim between the interfaces, groups of real number interpolations will have to be calculated both within one grid or the other and always to cross the interface. The advantage of a gridded system is the ability to quickly calculate transitions using counting and whole number math and systematic, repeating formulas, especially where this enables calculating relationships between non-adjacent cells without having to calculate states for intervening cells.

Because of the alignment of the rhombic dimensions with the square grid, there are a number of ways to interface between the two, creating a hybrid cellular space. A schema is proposed in the diagram below that provides a number of highly desirable characteristics for the composite space, and also illustrates a few of the challenges.

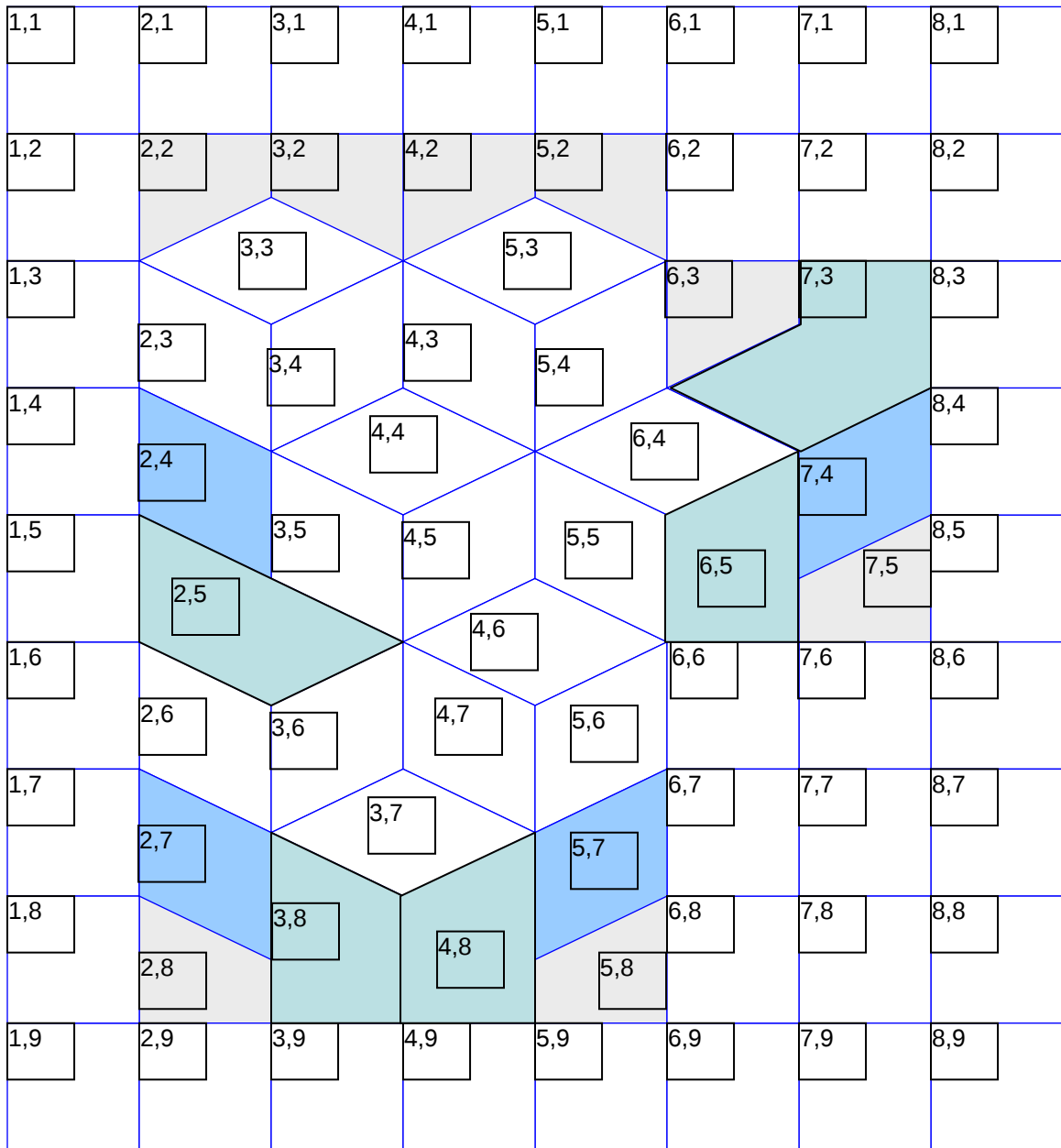
At first glance, the diagram shows two important things: (1) that the interface can be created through the repeated application a small number of simple steps, (2) that the *area* of the square grid can be maintained (that is, among the square cells, rhombic cells, and interface cells, all the cell coordinates that would have been in a homogeneous square grid of the same size are accounted for). The color coding scheme highlights one of the obvious challenges. The scheme shows: *normal* square and rhombic cells in white; *truncated* cells in gray; *augmented* cells in cyan; and *added* cells in blue.

For the truncated cells, a square cell that overlaps with a rhombic cell is considered to be only the part of the square cell that is exclusive of a rhombic cell. Conceptually parallel to the idea of being truncated, some of the truncated cells (the “points down” ones) only share a side with three different cells. The effect on implementation is that change of position in two different directions could lead to the same place.

For the augmented cells, a square cell that overlaps with a rhombic cell is subsumed in the rhombic cell. In a similar conceptual parallel, these cells share a side with more than four other cells, and may share multiple sides with one other cell or have single geometric sides that border two cells. This can be handled in two primary ways: (1) ignore one of the border cells so there is no legal cell to cell transition where there is a physical sense that there should be one, (2) add custom decision and management criteria for transitions from these cells.

The added cells are less problematic. They are simply additional rhombic cells that don’t align with the existing pattern that are created to aid in the interfacing process.

Across all the interface cells, there are challenges with numbering and transitions, but these are relatively easy to deal with through the repetitive application of integer based algorithms. Their most significant challenge, however, is that they cause noticeable problems with the *flow*, of the grid, which is the next topic.



an interfacing schema

As was earlier mentioned, the entire set of expected coordinates from the square grid are mappable to the hybrid grid, with a small number of difficulties and a reasonable number of reasonably easily handled special cases (This is especially true when you consider the typical application stated for this approach – the meshing of relatively large areas of the two systems to create a combined urban-open area that is tactically useful for wargaming. The number of normal cells grows geometrically with expansion of the area, while the number of interface cells grows only linearly, with the problem cells being a relatively small portion of those.) But the piecewise changes from cell to cell are not as important as the overall performance, especially over large distances and across multiple transitions – the flow of the hybrid grid. Using the base square grid as a referent, we will look at the overall horizontal and vertical flows.

The horizontal flow is the easiest to address and the simplest to evaluate. One can transition from the (1,x) to the eighth column cells in seven steps, the same as the square grid. Almost all of them are through a path that stays in the same row and most of them can be achieved with a repeated series of east moves, again the same as the square grid. Since the variations are small in number and size, this seems to be a very good fit. Again, going back to the typical use case, the existence of a few places where you have to move a little to the side to cross an area of rough ground does not seem to be incongruous.

The vertical flow has more challenges. The only paths through the rhombic grid that can maintain the number of crossing steps that the square grid has are ones that start near the vertical edges of the interface area. The minimum transit paths vertically through the center of the rhombic grid only cost about one more transition in ten, which is compatible with the distance loss of the squashed hexagonal grid, and isn't especially incongruous with the typical use case. Additionally, all the minimum vertical paths that don't run along the interface edge require some lateral movement. A large variation is not required, but it does affect a large number of paths.

When you couple the horizontal and vertical flow behaviors, an interesting characteristic emerges – there is an “easy” direction to transit the area, and a “hard” direction, that is, horizontal transition consistently requires less variation from the square grid performance than vertical. This behavior is not incongruous with many open, natural areas which have bias in the sloping and variation of the terrain, but it is not strongly compatible with all types of typical use case terrain (such as a groomed and manicured park in an urban area).

When we note that this is only a function of horizontal and vertical as displayed in the example and not all grids in general, we see yet another interesting emergent behavior. Rotational symmetry of the grid allows us to swap the vertical and horizontal of the sample without changing the performance parameters relative to the original horizontal and vertical – the easy way becomes the hard way, and vice versa. This property creates the opportunity to, within the same overall map, have subsections of rhomboid grid that have different *orientations*, which could be managed to impact that tactical characteristics of the terrain.

One additional emergent behavior of the approach is that swapping out top for bottom rhomboid configurations in the superposed hexagonal cells does not affect the long overall performance of interfaced gridded areas, but can cause additional local variations where the changes occur. Again, this behavior could be managed to support specific terrain effects that impact the tactical characteristics of the terrain.

As a final significant parameter of the hybrid grid, while it is not possible to easily integrate *every* possible combination of hexagonal and square natural areas, the interface is able to be changed in a regulated fashion at the edges in small increments. This means it would be relatively easy to integrate a wide variety of shapes for both types of sub grid.

In summary, the superposition of a rhombic grid over a base square grid provides the opportunity to leverage the advantages of gridded wargame battlespaces and enables optimization for urban and natural terrain in a piecewise fashion with a minimum of compromises.